Modeling VaR OF DAX INDEX USING GARCH MODEL

Matej Štalmach[1]

Abstract

The Box-Jenkins time series analysis rests on important concept as stationarity and residuals of ARMA models follows white noise. These concepts are insufficient for the analysis of financial time series. The paper proposes main characteristics of volatility in financial time series and general overview of most common models for time series modeling. This paper also outlines characteristics of DAX and SAX index's volatility and shows how to specify a composite conditional mean and variance model using GARCH(1,1) model. We finally apply GARCH methodology to estimate VaR and compare with other approach for DAX and SAX indices. In general those indices might represent development and variability of business sector in German and Slovak economy as well. The paper presents conditional model for volatility of economic growth and should be the basis for further investigation of mechanisms in the real economy in Slovakia as well as comparison with volatility of economic growth in Germany

Abstrakt

Metodológia analýzy časových radov podľa Box-Jenkins je založená na predpoklade stacionarity a predpoklade, že rezídua ARMA modelu nasledujú biely šum. Tieto predpoklady však nie sú často krát splnené pri analýze, modelovaní finančných časových radov. Táto práca približuje základne charakteristiky volatility finančných časových radov a prináša prehľad jedného z najpoužívanejších modelov na štatistický opis časového radu. Charakteristika volatility, ako aj špecifikácia kompozitného modelu podmienenej strednej hodnoty AR(1) a podmienenej variancie GARCH(1,1), sú demonštrované na časovom rade DAX a SAX indexu. Na koniec je metodológia GARCH aplikovaná

[1] Ing. Mgr. Matej Štalmach, Tatrabanka a.s.,Černyševského 50 , 851 01 Bratislava, tel. 0914373708; Fakulta hospodárskej informatiky EU Dolnozemská cesta 1/b 852 35 Bratislava, e-mail: matej.stalmach@gmail.com

na odhad VaR a konfrontovaná s ostatnými bežnými metódami výpočtu VaR. Indexy DAX a SAX reprezentujú pri istej miere zovšeobecnenia vývoj a variabilitu ekonomiky Nemecka respektíve Slovenska, najmä jej podnikateľskej sféry. Práca prináša model podmienenej volatility ekonomického vývoja a je základ ďalšieho skúmania mechanizmov v reálnej ekonomike na Slovensku ako aj porovnanie s Nemeckom

Keywords: DAX Index, SAX Index, volatility, GARCH

Kľúčové slová: DAX Index, SAX Index, volatilita, GARCH

Introduction

Financial time series can be characterized by three separate components – seasonality, trend and fluctuations around components. The trend is mostly the strategic fundaments for a given variable time series especially from a longer time perspective. Fluctuations rate in financial variables is called volatility, which is the square root of the variance.

1. FINANCIAL SERIES

1.1 Volatility of financial time series

Typically the volatility has these features:

- **Volatility clustering**: in yield occur frequently phenomena that high volatility is followed by high volatility and low by low volatility, thus the volatility has the autocorrelation characteristics. That is why it is interesting to use GARCH model for modeling the distribution of income, even when the model cannot explain this phenomenon.
- **Leverage effect**: refers to the well-established relationship between stock returns and both implied and realized volatility: volatility increases when the stock price falls. A standard explanation ties the phenomenon to the effect a change in market valuation of a firm's equity has on the degree of leverage in its capital structure, with an increase in leverage producing an increase in stock volatility.
- Volatility is **evolving continuously** over time, volatility jumps are continuous.
- Volatility **not diverges to infinity**, but in the long term is often stationary.

1.2 Time series - introduction

1.2.1 Standard time series models:

The class of ARMA models is the most widely used for the prediction of second-order stationary processes[2]. It uses an iterative six-stage scheme summarized by Francq, Zakoian (2010):

(i) a priori identification of the differentiation order d (or choice of another transformation);
(ii) a priori identification of the orders p and q;
(iii) estimation of the parameters
(iv) validation;
(v) choice of a model;
(vi) prediction

A number of sources describe ARMA; among others Bollerslev (2011):

y_t=E$(y_t|\Omega_{t-1}) + \varepsilon_t$
E$(y_t|\Omega_{t-1}) = \mu_t(\theta)$
$Var(y_t|\Omega_{t-1}) = E(\varepsilon_t^2|\Omega_{t-1}) = h_t(\theta) = \sigma^2$
ARMA(p,q) model:
$\mu_t(\theta) = \varphi_0 + \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + \theta_1\varepsilon_{t-1} + \cdots + \theta_q\varepsilon_{t-q}$

We may derive main statistic description of time series

Conditional mean $\mu_t(\theta)$:	varies with Ω_(t-1)
Conditional variance $h_t(\theta)$:	constant
Unconditional mean $\mu(\theta)$:	constant
Unconditional variance $h(\theta)$:	constant

1.2.2 ARCH – AutoRegressive Conditional Heteroskedasticity:

Modeling financial time series is a complex problem. This complexity is crucial even we transform non-stationary price time series into series of return which had seemed to be stationary and followed by white noise. Francq, Zakoian outlines empirical findings why we should improve model as Engle (1982) did; presented by Bollereslev (2011):

[2] To simplify presentation, we do not consider seasonal series, for which SARIMA models can be considered This methodology is proposed by Box et al. (2008), 4th edition of famous Box, Jenkins (1994)

$y_t = E(y_t|\Omega_{t-1}) + \varepsilon_t$
$E(y_t|\Omega_{t-1}) = \mu_t(\theta)$
$\mathrm{Var}(y_t|\Omega_{t-1}) = E(\varepsilon_t^2|\Omega_{t-1}) = h_t(\theta)$
ARCH(q) model:
$h_t = \omega + \alpha_1\varepsilon_{t-1}^2 + \cdots + \alpha_q\varepsilon_{t-q}^2$

Major improvement is to consider heteroscedasticity and suggest model variance with lagged

Conditional mean $\mu_t(\theta)$:	varies with Ω_(t-1)
Conditional variance $h_t(\theta)$:	varies with Ω_(t-1)
Unconditional mean $\mu(\theta)$:	constant
Unconditional variance $h(\theta)$:	constant

1.2.3 GARCH - Generalized ARCH:

Since the articles by Engle (1982) on ARCH (autoregressive conditionally heteroscedastic) processes a large variety of papers have been devoted to the statistical inference of these models, any of them is difficult to understand and compute. Complexity is proportional with number of parameters so Bollerslev (1986) improved model with lagged squared innovations and dramatically decreased time of inference.

$y_t = E(y_t|\Omega_{t-1}) + \varepsilon_t$
$E(y_t|\Omega_{t-1}) = \mu_t(\theta)$
$\mathrm{Var}(y_t|\Omega_{t-1}) = E(\varepsilon_t^2|\Omega_{t-1}) = h_t(\theta)$
GARCH(p,q) model:
$h_t = \omega + \alpha_1\varepsilon_{t-1}^2 + \cdots + \alpha_q\varepsilon_{t-q}^2 + \beta_1 h_{t-1} + \cdots + \beta_p h_{t-p}$

The simple GARCH(1,1) model often works very well $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$

Conditional mean $\mu_t(\theta)$:	varies with Ω_(t-1)
Conditional variance $h_t(\theta)$:	varies with Ω_(t-1)
Unconditional mean $\mu(\theta)$:	constant
Unconditional variance $h(\theta)$:	constant

2. APPLICATION GARCH MODEL IN RISK MANAGEMENT

The recent financial crisis and its impact on the broader economy underscore the importance of financial risk management in today's world. At the same time, financial products and investment strategies are becoming increasingly complex. Today, it is more important than ever that risk managers possess a sound understanding of mathematics and statistics in order to ensure that the business model has fewer surprises. Volatility is a measure which is by definition about variability in general and applied to return series of financial instrument it is about uncertainty of future profits. GARCH models led to a fundamental change to the approaches used in finance, through an efficient modeling of volatility (or variability) of the prices of financial assets.

The aim of the paper is to provide example how use GARCH model in practical finance management. It is worth mention that the most important decision is on senior executives who have limited time and knowledge to make a decision what explained use of GARCH (1,1) model. Use of this model could be explained by the fact that they are still simple enough to be usable in practice.

2.1 Specify and estimate Conditional Mean and Variance Models using GARCH model

The German Stock Index is a total return index of 30 selected German blue chip stocks traded on the Frankfurt Stock Exchange. The equities use free float shares in the index calculation. The DAX has a base value of 1,000 as of December 31, 1987. As of June 18, 1999 only XETRA equity prices are used to calculate all DAX indices

This example shows how to estimate a composite conditional mean and variance model using GARCH (1,1) for variance and AR(1) for mean. We use software Matlab statistical toolbox and follow algorithm from Matlab’s tutorial[3]:

Step 1. Load the data and specify the model: Data was prepared from http://finance.yahoo.com/q/hp?s=^GDAXI+Historical+Prices and contained price time series of DAX from 3.1.2000 to 28.3.2014 which was transformed to the continuously compounded returns series and the same process was done with data obtained from

[3]http://www.mathworks.com/help/econ/conditional-mean-and-variance-model-for-nasdaq-returns.html#zmw57dd0e26313

http://www.bsse.sk/Obchodovanie/Indexy/IndexSAX.aspx contained price time series of SAX from 7.1.2000 to 28.3.2014

P_t means price of DAX/SAX index in time t

r_t means return of DAX/SAX index from time t-1 to t

$$P_t = (1 + r)P_{t-1}$$

$$P_t = \lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^n P_{t-1}$$

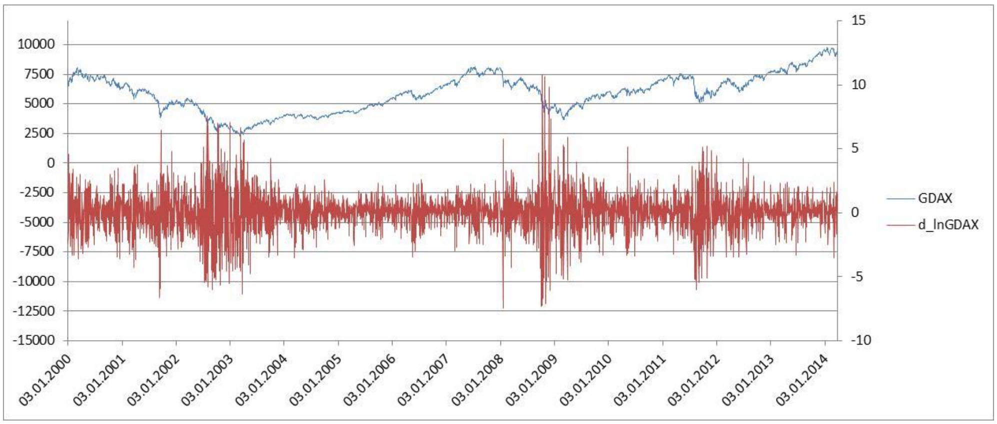
$$P_t = \lim_{n\to\infty} \left(\left(1 + \frac{1}{n/r}\right)^{n/r}\right)^r P_{t-1} = e^{r_t} * P_{t-1}$$

$$r_t = \ln \frac{P_t}{P_{t-1}} = \ln P_t - \ln P_{t-1}$$

Graphical representation of DAX returns in figure 1 outlines all characteristics of volatility mentioned in chapter 1.1. SAX returns does not show the characteristics explicitly:

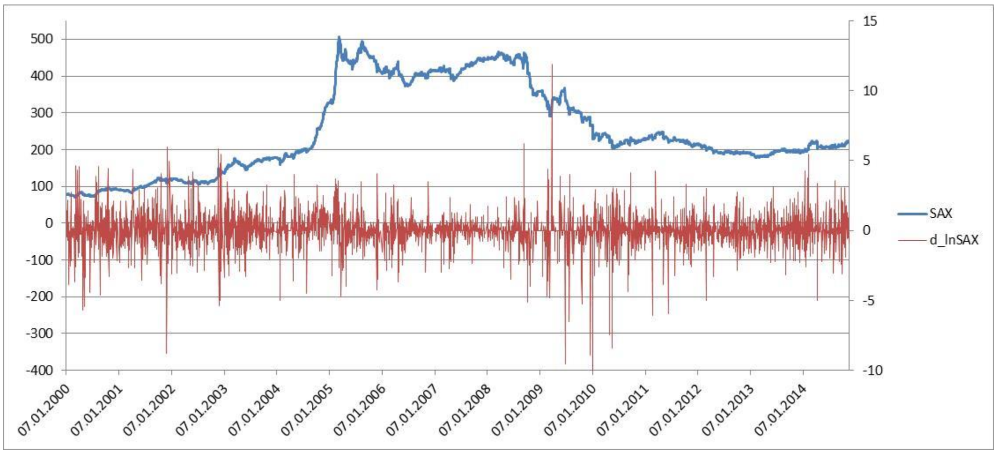
- **Volatility clustering**: after huge crisis volatility remains high; in “good” times between crisis stays low
- **Leverage effect**: with decrease of DAX price increases volatility of returns
- **evolving continuously:** no jumps
- **not diverges to infinity** even during Global financial crises in 2008 called the worst financial crisis since the Great Depression of the 1930s

Figure 1: Price [GDAX] and return [d_lnGDAX] of DAX index



Source: http://finance.yahoo.com/q/hp?s=^GDAXI+Historical+Prices prepared by author

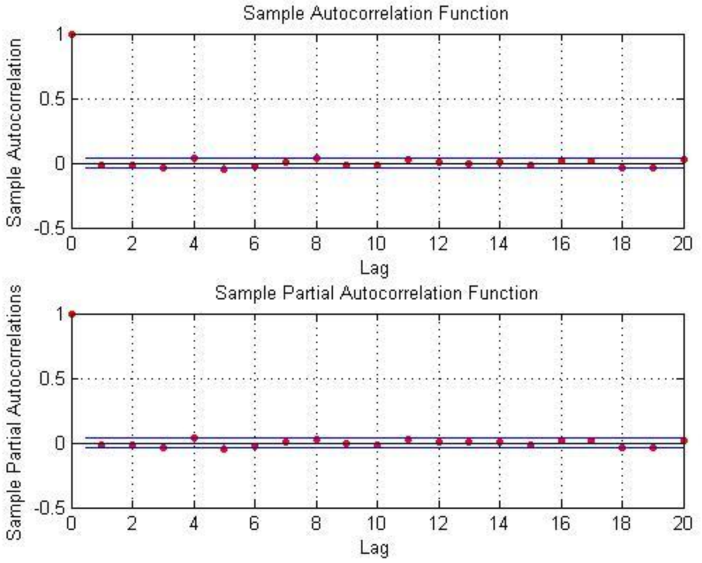
Figure 2: Price [SAX] and return [d_lnSAX] of SAX index



Source: http://www.bsse.sk/Obchodovanie/Indexy/IndexSAX.aspx prepared by author

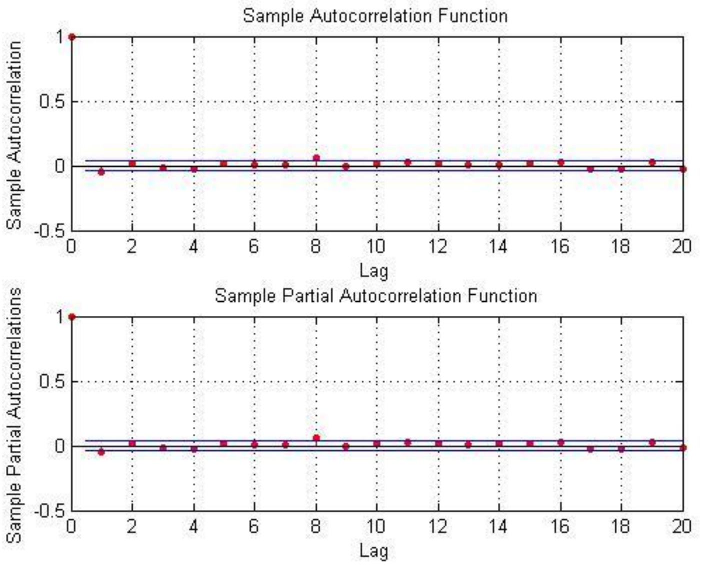
Step 2. Check the series for autocorrelation: ACF or PACF in Figures 3 and 4 are not suggested significant AR or MA process for DAX or SAX returns so we continue with step 3.

Figure 3: ACF and PACF of DAX returns time series



Source: prepared by author

Figure 4: ACF and PACF of SAX returns time series

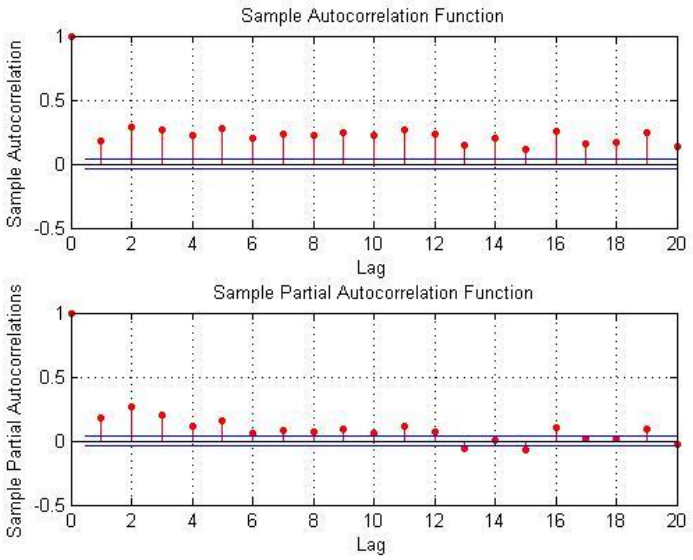


Source: prepared by author

Step 3. Test the significance of the autocorrelations: The null hypothesis that all autocorrelations are 0 up to lag 5 is rejected (h = 1) p = 5.5839e-04 for DAX returns. This test is rejected also for SAX returns however p is much higher (0.0157)

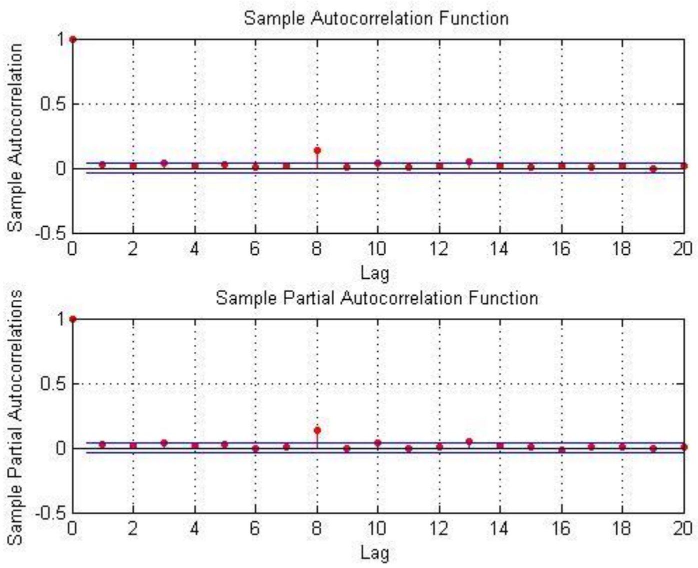
Step 4. Check the series for conditional heteroscedasticity: Figure 5 shows the sample ACF and PACF of the squared return series. The autocorrelation functions show significant serial dependence. SAX returns does not offer so much certainty on Figure 6.

Figure 5: ACF and PACF of squared DAX returns time series



Source: prepared by author

Figure 6: ACF and PACF of squared SAX returns time series



Source: prepared by author

Step 5. Test for significant ARCH effects: Conduct an Engle's ARCH test. Test the null hypothesis of no conditional heteroscedasticity against the alternative hypothesis of an ARCH model with two lags (which is locally equivalent to a GARCH(1,1) model). Result is H = 0 by p = 0. Test rejected null hypothesis for DAX index also for SAX (but p of test equals 0.1078 for SAX)

Step 6. Specify a conditional mean **and variance model:** In Table 1 is specified parameters of the AR(1) model for the conditional mean of the DAX and SAX returns, and the GARCH(1,1) model for the conditional variance. This is a model of the form:

$r_t = \varphi_0 + \varphi_1 r_{t-1} + \varepsilon_t$ [AR(1) model] where

$\varepsilon_t = \sigma_t z_t$

${\sigma_t}^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta{\sigma_{t-1}}^2$ [GARCH(1,1) model]

z_t - is an independent and identically distributed standardized Gaussian process.

Table 1: parameters of AR(1) for mean and GARCH(1,1) for variance Gaussian residuals

Model	Parameter	DAX Index			SAX Index		
		Value	Standard Error	t Statistic	Value	Standard Error	t Statistic
AR(1)	Constant	0.073	0.019	3.922	0.021	0.018	1.168
	AR{1}	-0.025	0.019	-1.327	-0.072	0.016	-4.501
Model	Parameter	Value	Standard Error	t Statistic	Value	Standard Error	t Statistic
GARCH(1,1)	Constant	0.023	0.004	5.933	0.009	0.001	10.446
	GARCH{1}	0.902	0.008	114.314	0.970	0.001	885.640
	ARCH{1}	0.088	0.007	12.128	0.026	0.001	27.142

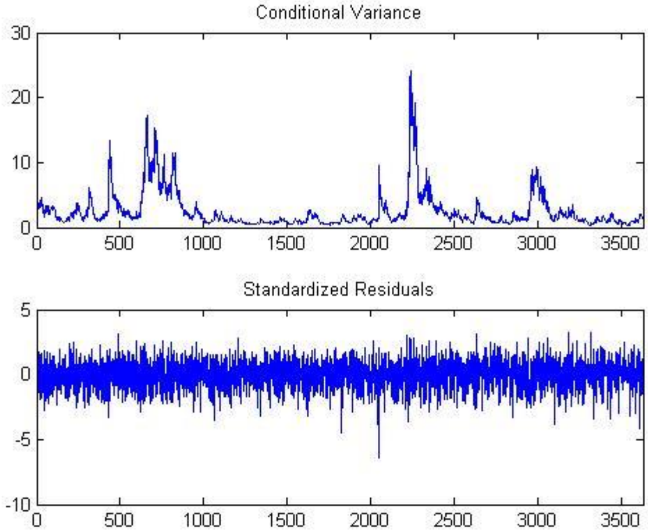
Source: prepared by author

Step 7. Infer the conditional variances and residuals: Figures 7 shows that the conditional variance of DAX return increases after observation 750, 2 250, 3 000. This corresponds to the increased volatility seen in the original return series at the end of 2002 - the Internet bubble bursting; at the end of 2008 – Global Financial Crisis (The active phase of the crisis, which manifested as a liquidity crisis); 2011 - fears of contagion of the European sovereign debt crisis to Spain and Italy

From part of Figure 7 we may conclude that the standardized residuals have more large values (larger than 2 or 3 in absolute value) than expected under a standard normal distribution. This suggests a Student's t distribution might be more appropriate for the innovation distribution – next step

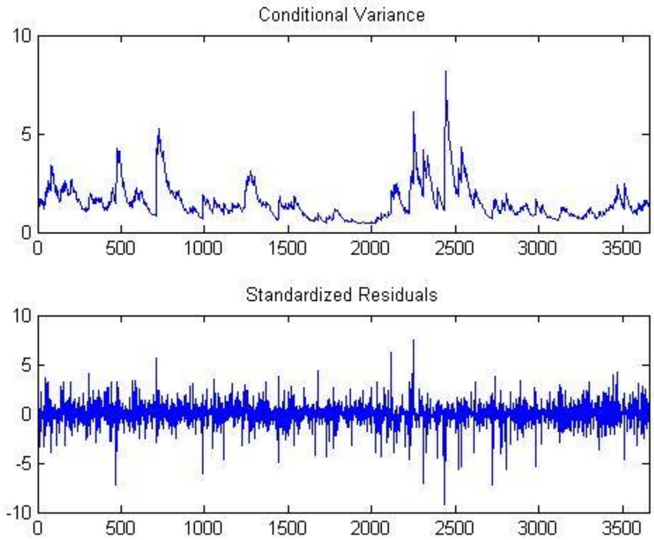
Figures 8 shows that the conditional variance of SAX return increases after observation 750, 2 250, 2 500. It is hard to say whether this volatility increses are explained with shock on global market and it is worth to say that represet dates in original data series are in March 2003, April 2009 and April 2010 so lagged few month after shock on global markets.

Figure 7: Conditional variance and standard residuals infers from AR(1)/GARCH(1,1) model for DAX



Source: prepared by author

Figure 8: Conditional variance and standard residuals infers from AR(1)/GARCH(1,1) model for SAX



Source: prepared by author

Step 8. Fit a model with a t innovation distribution: In Table 2 is specified parameters of the AR(1) model for the conditional mean of the DAX and SAX returns, and the GARCH(1,1) model for the conditional variance. This is a model of the form

$r_t = \varphi_0 + \varphi_1 r_{t-1} + \varepsilon_t$ [AR(1) model]

where

$\varepsilon_t = \sigma_t z_t$

$\sigma_t{}^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}{}^2$ [GARCH(1,1) model]

z_t - is an independent and identically distributed Student's *t* process.

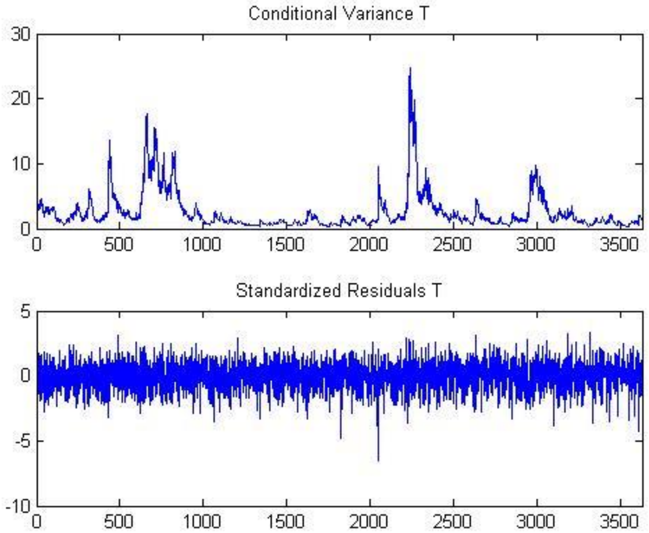
Table 2: parameters of AR(1) for mean and GARCH(1,1) for variance students residuals

Model	Parameter	DAX Index			SAX Index		
		Value	Standard Error	t Statistic	Value	Standard Error	t Statistic
AR(1)	Constant	0.084	0.018	4.615	0.039	0.012	3.395
	AR{1}	-0.021	0.018	-1.132	-0.067	0.013	-5.229
	DoF	10.197	1.573	6.481	2.343	0.111	21.072
Model	Parameter	Value	Standard Error	t Statistic	Value	Standard Error	t Statistic
GARCH(1,1)	Constant	0.017	0.005	3.573	0.128	0.039	3.245
	GARCH{1}	0.908	0.010	95.087	0.859	0.018	48.286
	ARCH{1}	0.087	0.009	9.193	0.141	0.041	3.448
	DoF	10.197	1.573	6.481	2.343	0.111	21.072

Source: prepared by author

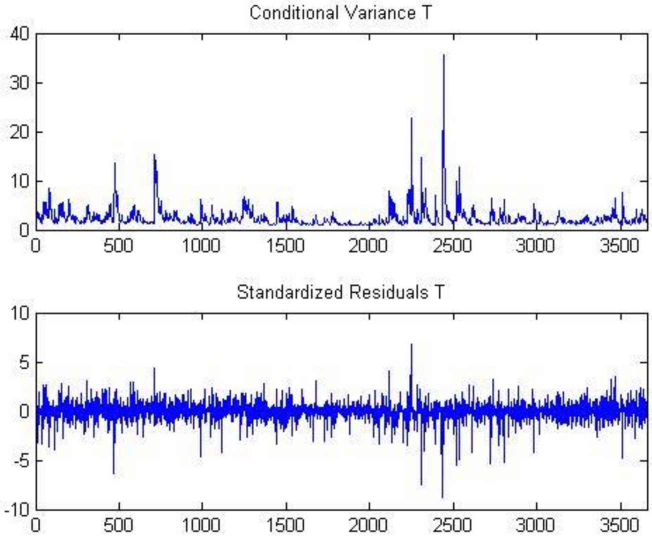
Graph of Conditional variance and standard residuals of this model in Figure 5 follows the same path as for normal distributed innovations.

Figure 9: Conditional variance and standard residuals infers from AR(1)/GARCH(1,1) model for DAX



Source: prepared by author

Figure 10: Conditional variance and standard residuals infers from AR(1)/GARCH(1,1) model for SAX



Source: prepared by author

Step 9. Compare the model fits: Table 3 compare the two model fits (Gaussian and t innovation distribution) using the Akaike information criterion (AIC) and Bayesian information criterion (BIC).

Table 3: AIC and BIC criterion for suggested models

Stat	DAX Index		SAX Index	
	Gaussian	t	Gaussian	t
AIC	12 217	12 162	11 424	9 844
BIC	12 248	12 199	11 455	9 881

Source: prepared by author

The second model has six parameters compared to five in the first model (because of the t distribution degrees of freedom). Despite this, both information criteria favor the model with the Student's t distribution and it is valid for DAX and SAX returns as well. The AIC and BIC values are smaller for the t innovation distribution

As the last step is a test decision for the null hypothesis that the data of ressiduals comes from a normal distribution, using the Jarque-Bera test. Despite of the fact that we dramatically decreased jb statistics, we calculated the result h is 1 and the test rejects the null hypothesis at the 5% significance level.

2.2 Value at Risk

2.2.1. Definition and computing of Value at risk

(VaR) is the most widely used risk measure in financial industry. In 1993, the business bank JP Morgan publicized its estimation method, RiskMetrics, for the VaR of a portfolio. VaR is now an indispensable tool for banks, regulators and portfolio managers. Hundreds of academic and nonacademic papers on VaR may be found at http://www.gloriamundi.org/ also a lot of books were written about VaR, some of them became bestsellers i.e Value at risk – the new benchmark for managing finacial risk by Jorion (1996) provided the first commprehensive description of value at risk. It quicly established itself as an indipensable reference on VaR and has been called ‘Industry standard’. Value at Risk theory and practise by Holton (2003) offers almost pure academic approuch with extensive theory of risk measure and metric. Measuring market Risk by Dowd (2005) overviewed of the state of the art in market risk measurement. VaR summurizes the worst loss over a target horizon with a given level of confidence. Mathematical definition: if L is the loss of a portfolio, then $VaR_\alpha(L)$ is the level α-quantile, i.e.

$$VaR_\alpha(L) = \inf\{l \in \mathbb{R}: P(L > l) = (1 - \alpha)\}$$

For practical demonstration is used 1 day 99% VaR of DAX index

$$VaR_{0.99}(L) = \inf\{l \in \mathbb{R}: P(L > l) = 0.01\}$$

Many papers could be found with the keywords calculation of VaR. Many of them may be at http://gloria-mundi.com. Basic clasification by Dowd (2007):

- **Non-parametric approaches**:
  - Basic historical simulation
  - Bootstrapped historical simulation
  - Historical simulation using non-parametric density estimation
- **Parametric approaches**:
  - Unconditional distribution
  - Conditional distribution
- **Monte Carlo simulation**

2.2.2. Compute VaR using GARCH(1,1) and compare with other methods

This example shows calculation 1day 99% VaR of DAX Index[4] using a composite conditional mean and variance model GARCH (1,1) for variance and AR(1) for mean from previous example. It is worth to calculate VaR using other basic methods to evaluate accuracy of GARCH model. For calculation VaR is used floating window of 2 000 historical observation of DAX index daily return in fact loss.

VaR99_HS VaR is calculated as 20th worst loss from the 2 000 observations.

VaR99_ND VaR is calculated as 99th percentil of normal distribution with estimated mean and variance

$$\mu = \sum_{i=1}^{2\,000} \mathrm{d\_lnGDAX}_i$$

$$\sigma = \left(\frac{1}{1\,999} \sum_{i=1}^{2\,000} (\mathrm{d\_lnGDAX}_i - \mu)^2 \right)^{0.5}$$

$$\mathrm{VaR99\_ND} = \mu - \mathrm{NormInv}(0.99) * \sigma$$

μ	sample mean
σ	corrected sample standard deviation
NormInv	computes the inverse of the standard normal cdf

[4] Methods of calculation VaR are described by DAX Index but it is valid in general also for SAX.

VaR99_AR1 VaR is Calculated as 99th percentil of normal distribution with unconditional mean from AR(1) process and estimated variance

$\text{est}(\text{d\_lnGDAX})_t = \varphi_0 + \varphi_1 * \text{d\_lnGDAX}_{t-1}$

$\text{VaR99\_AR1} = est\left(\text{d}_{\text{lnGDAX}_t}\right) - \text{NormInv}(0.99) * \sigma$

est(d_lnGDAX) forecast of profit/loss generated by AR(1) model

φ_0, φ_1 parameters of AR(1) process

NormInv computes the inverse of the standard normal cdf

VaR99_G11_nd VarR is calculated as 99th percentil of normal distribution with parameters specified by a conditional mean and variance model

$\text{est}(\text{d\_lnGDAX})_t = \varphi_0 + \varphi_1 * \text{d\_lnGDAX}_{t-1}$

$variance(\text{d\_lnGDAX})_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta * variance(\text{d\_lnGDAX})_{t-1}$

$VaR99\_G11_{nd} = \text{est}(\text{d\_lnGDAX})_t - \text{NormInv}(0.99) * \text{est}(variance(\text{d\_lnGDAX})^{0.5})$

est(d_lnGDAX) forecast of profit/loss generated by AR(1) model with conditional variance forecasted by GARCH(1,1) model

variance(d_lnGDAX) forecast of variance by GARCH(1,1) model

$\varphi_0, \varphi_1, \omega, \alpha, \beta$ parameters of common AR(1) process with conditional variance estimated by GARCH(1,1) model

NormInv computes the inverse of the standard normal cdf

VaR99_G11_sd VarR is calculated as as 99th percentil of student distribution with parameters specified by a conditional mean and variance model

$$\text{d\_lnGDAX}_t = \varphi_0 + \varphi_1 * \text{d\_lnGDAX}_{t-1}$$

$$variance(\text{d\_lnGDAX})_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta * variance(\text{d\_lnGDAX})_{t-1}$$

$$VaR99\_G11\_sd = est\left(\text{d}_{\text{lnGDAX}_t}\right) - \text{TInv}(0.99) * \frac{\sqrt{variance(\text{d\_lnGDAX})}}{\sqrt{\frac{df}{df-2}}}$$

est(d_lnGDAX) forecast of profit/loss generated by AR(1) model with conditional variance forecasted by GARCH(1,1) model

variance(d_lnGDAX) forecast of variance by GARCH(1,1) model

$\varphi_0, \varphi_1, \omega, \alpha, \beta$ parameters of common AR(1) process with conditional variance estimated by GARCH(1,1) model

TInv Student's t inverse cumulative distribution function

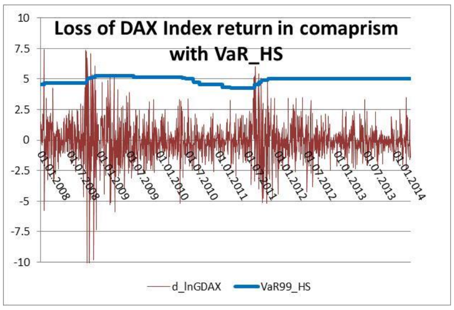
df degree of freedom

These calculations of VaR methods is used to estimate 1 day 99% VaR using 2 000 historical observation each working day from the 12th of November 2007 to the 28th of March 2014, it is 1 631 working days. To demonstrate accuracy of each method is calculated number of situation when estimated VaR is more then realized loss; it is called the bridge.

Results of estimation VaR of DAX returns are shown by figure 11, figure 12 and figure 13. For all methods situation when realized loss was over VaR is correleted with failor of Lehman Brothers in September of 2008 and European sovereign debt crisis before all EU countries agreed to expand the EFSF by creating certificates that could guarantee up to 30% of new issues from troubled euro-area governments. Table 4 provide basicoverview of accurancy of each method. From 1 631 estimation of VaR by historical simulation only 18 times occurred sitution when real loss was higher. But difference between VaR and time series of gain and loss is significant deep and this method shoul be noticed many banker as conservative. Unconditional parametric method are weak in terms of number of the bridges and difference is close to historical simulation. Path of profit/loss and GARCH(1,1) VaR time series shows correlation and by improvement with student distribution number of the bridegs are closer to theoretical expected values 16.3.

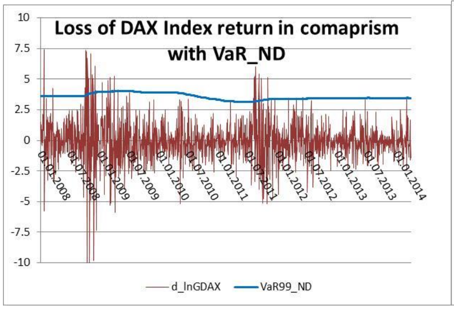
Results of estimation VaR of SAX returns are shown by figure 14, figure 15 and figure 16 and conclusion from previos paragrph are almost applicable as well. Only all methods of calcualtion are less effective and resualts are more biased due the fact that all assumptions were not met. Table 4 describes whole picture that Historical simulations is the nearest to theretical expected value of the brideges however path of estimated loss is far from realized losses on the other hand GARCH model with Students improvement has still accteble results and realized and estimated loss are the closest.

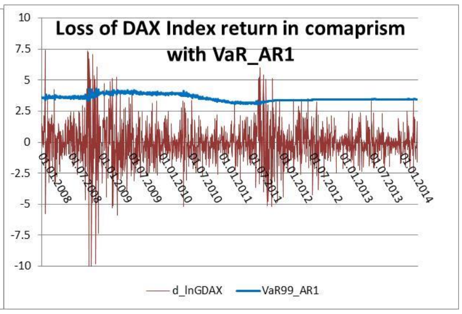
Figure 11: Observed losses of DAX index compare to estimated 1 day 99% VaR calculated by non-parametric method - historical simulation



Source: prepared by author

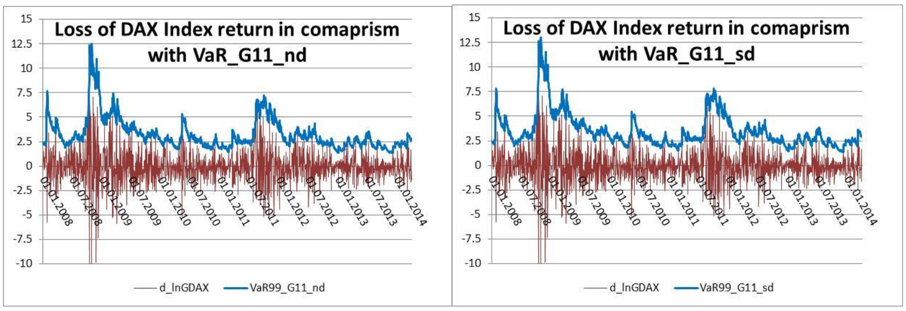
Figure 12: Observed losses of DAX index compare to estimated 1 day 99% VaR calculated by unconditional parametric methods – normal distribution, AR1 process





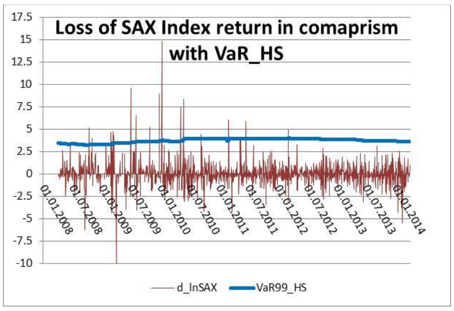
Source: prepared by author

Figure 13: Observed losses of DAX index compare to estimated 1 day 99% VaR calculated by conditional parametric methods – GARCH (1,1) process



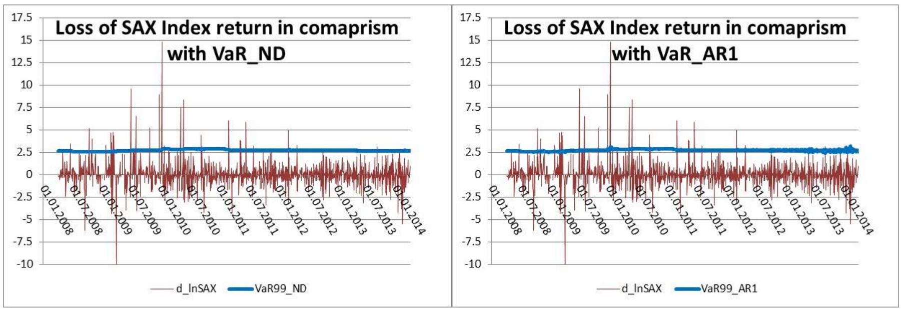
Source: prepared by author

Figure 14: Observed losses of SAX index compare to estimated 1 day 99% VaR calculated by non-parametric method - historical simulation



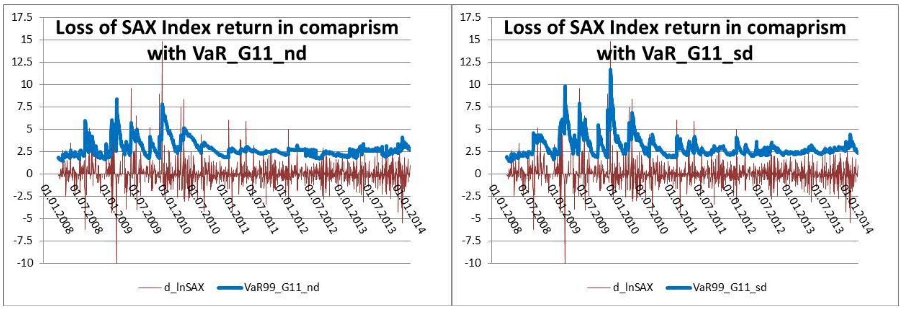
Source: prepared by author

Figure 15: Observed losses of SAX index compare to estimated 1 day 99% VaR calculated by unconditional parametric methods – normal distribution, AR1 process



Source: prepared by author

Figure 16: Observed losses of SAX index compare to estimated 1 day 99% VaR calculated by conditional parametric methods – GARCH (1,1) process



Source: prepared by author

Table 4: Count of situations when realized loss was above estimated VaR

VaR Method	No of bridge	
	DAX	SAX
# of observation	1 631	1497
1% of observation	**16.3**	**15.0**
VaR99_HS	18	18
VaR99_ND	38	29
VaR99_AR1	38	30
VaR99_G11_nd	33	27
VaR99_G11_sd	26	22

Source: prepared by author

Conclusion

The subject of the article is to analyse the necessity of adopting conditional volatility model for returns as shows Figure 1. Paper provided the basic demonstrations of theoretical result and illustrated the main techniques with numerical examples. Figure 7 and 9 proof that GARCH(1,1) is robust enough to model conditional volatility of DAX revenues despite the fact that ressidual does not follow Gaussian distribution. Student's t distribution of innovation improves model sligtly. GARCH (1,1) is easy to calculate using Matlab, correct volatility on average, exaggerates volatility-of-volatility. Example of VaR calculating by using GARCH(1,1) shows a substantial gain in accuracy. VaR by GARCH(1,1) estimeted number of situation when observed loss is higher than estimated VaR worse than historical simulation however path of this estimation is much closer to real observations.

Conclusion of modeling volatility of capital market is more useful for German economy according existing and pretty dynamic capital market and model of volatility offered path of instability for whole economy especially in times of global shocks as internet bubble, failure of Lehman Brothers and sovereign debt crises. Volatility of growth in Slovak economy might be still represented by SAX index but results are not sufficient supported by significance of estimated parameters. This paper does not research whether capital market in Slovakia is sufficient or not however general opinion it is not. So to understand behavior of Slovak economy from perspective of entrepreneur might be worth to dive deeper to raw data from microeconomics perspective (number of failures or bankrupts of companies) and consider relations between German DAX due better indication of global crisis. Slovak data seems to be lagged which indicates that Slovak policy might have applicable alert how the market react to global crises but without opportunity to change the destiny.

Acknowledgement

This contribution is the result of the project VEGA (2/0160/13) Financial stability and sustainable economic growth of Slovakia in global economy (Finančná stabilita a udržateľnosť hospodárskeho rastu Slovenska v podmienkach globálnej ekonomiky)

REFERENCES

[1] Bloomberg. (2014). *Deutsche Boerse AG German Stock Index DAX*http://www.bloomberg.com/quote/DAX:IND [accessed 1.5.2014]

[2] Burza cenných papierov v Bratislave: Slovenský akciový index SAX http://www.bsse.sk/Obchodovanie/Indexy/IndexSAX.aspx [accessed 5.11.2014

[3] Bollerslev T. (2011). *ARCH and GARCH models* http://public.econ.duke.edu/~boller/Econ.350/talk_garch_11.pdf [accessed 1.5.2014]

[4] Francq, Ch; Zakoian J. M. (2010). *GARCH Models Structure, Statistical Inference and Financial Applications*. Lille: John Wiley & Sons Ltd, 2010.

[5] Holton A. (2003). *Value-at-risk: Theory and Practice*. New York: Academic Press, 2003.

[6] Jorion. P. (2006). *Value at Risk, 3rd Ed.: The New Benchmark for Managing Financial Risk*. New York: McGraw Hill Professional, 2006.

[7] Kevin D. (2007). *Measuring Market Risk*. West Sussex: Wiley & Sons, 2007.

[8] MathWorks (2014). *Specify Conditional Mean and Variance Models*http://www.mathworks.com/help/econ/conditional-mean-and-variance-model-for-nasdaq-returns.html [accessed 15.4.2014]

[9] Yahoo Finance (2014). *DAX (^GDAXI) – XETRA* http://finance.yahoo.com/q/hp?s=^GDAXI+Historical+Prices [accessed 15.4.2014]

Kontakt

Ing. Mgr. Matej Štalmach

University of Economics in Bratislava

Faculty of Economic Informatics

Dolnozemská cesta 1/b

852 35 Bratislava 5

Slovakia

e-mail: matej.stalmach@gmail.com

mobile: 0914373708